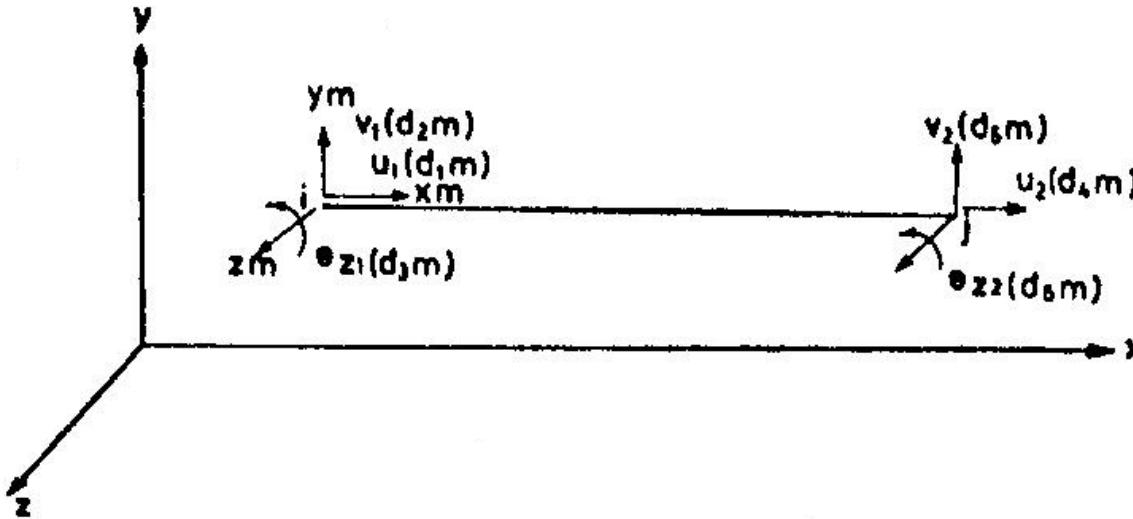


Two dimensional Beam :

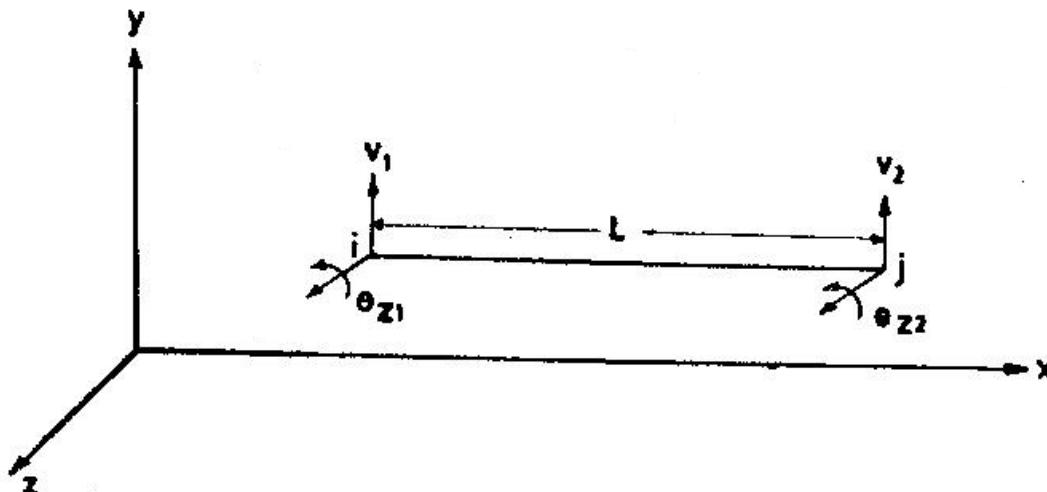


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Two dimensional beam element with six degrees of freedom

The degrees of freedom at the nodes are

$$\{d_m\}^T = \{d\}^T = [u_1 \ v_1 \ \theta_{z1} \ u_2 \ v_2 \ \theta_{z2}]$$



$$\theta = \frac{dv}{dx}$$

Two-dimensional beam element with four degrees of freedom

variation of v will be cubic and can be expressed as

$$v = \alpha_1 + \alpha_2 x + \alpha_3 x^2 + \alpha_4 x^3 \quad (7.20(a))$$

or in natural co-ordinates as

$$v = \alpha_1 L_1^3 + \alpha_2 L_2^3 + \alpha_3 L_1^2 L_2 + \alpha_4 L_1 L_2^2 \quad (7.20(b))$$

It may be noted that all the terms in Eq. (7.20 (b)) are cubic and if they are not, they can be made so by the relation $L_1 + L_2 = 1$.

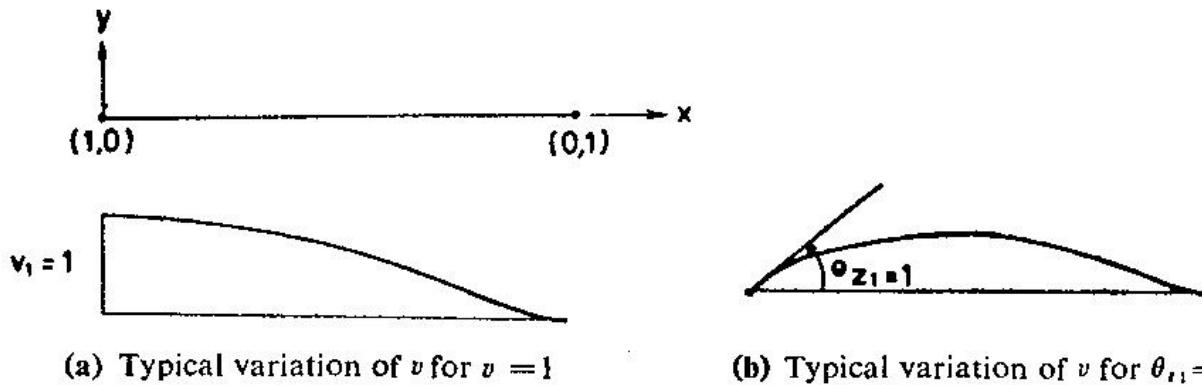


Fig. 7.7 Displacement variation for a two-dimensional beam Element

$$v = [N_1 \ N_2 \ N_3 \ N_4] \{d\}$$

$$\{d\}^T = [v_1 \ \theta_{z1} \ v_2 \ \theta_{z2}]$$

$$v = \alpha_1 L_1^3 + \alpha_2 L_2^3 + \alpha_3 L_1^2 L_2 + \alpha_4 L_1 L_2^2$$

$$\frac{dv}{dx} = \frac{1}{L} [3\alpha_2 L_2^2 + \alpha_3 L_1^2 + 2\alpha_4 L_1 L_2]$$

$$- 3\alpha_1 L_1^2 - 2\alpha_3 L_1 L_2 - \alpha_4 L_2^2]$$

Evaluation of N_2 : (Fig. 7.7(b))

At node i , $v_1=0$, $\theta_{z1}=1$ and at node j , $v_2=0$, $\theta_{z2}=0$ (7.26)

Substituting these values in Eqs 7.20b and 7.22 we get

$$\alpha_1=0, \alpha_2=0, z_3=L, \alpha_4=0 \quad (7.27)$$

Hence,

$$v=L_1^2 L_2 L$$

Thus,

$$N_2=L_1^2 L_2 L \quad (7.28)$$

Similarly the other components, N_3 and N_4 can be evaluated.

$$\text{Thus } [N]=[L_1^2(3-2L_1) \quad L_1^2 L_2 L \quad L_2^2(3-2L_2) \quad -L_1 L_2^2 L] \quad (7.29)$$

It can be shown from simple theory of bending that,

$$\epsilon_x = -y \frac{d^2 v}{dx^2} \quad (7.30)$$

$$\epsilon_y = -x \frac{d^2 v}{dy^2} \quad (7.30)$$

$$\sigma_x = -E y \frac{d^2 v}{dx^2} \quad (7.31)$$

$$\text{and bending moment } M = \int_{-h/2}^{h/2} \sigma_x y \cdot b \cdot dy \quad (7.32)$$

Substituting Eq. 7.31 in Eq. 7.32, we get

$$M = -EI_z \frac{d^2 v}{dx^2} \quad (7.33)$$

$$\text{where } I_z = \text{moment of inertia of the section} = \int_{-h/2}^{h/2} y^2 b \cdot dy \text{ about the local } z_m \text{ axis} \quad (7.34)$$

$$\frac{d^2v}{dx^2} = \frac{1}{L^2} [(6 - 12L_1) L - (2L_2 - 4L_1) (6 - 12L_2) \\ L(4L_2 - 2L_1)] \{d\} \quad (7.35)$$

$$M = -\frac{EI_z}{L^2} [(6 - 12L_1) L - (2L_2 - 4L_1) (6 - 12L_2) \\ L(4L_2 - 2L_1)] \{d\} \quad (7.36)$$

$$[C] = EI_z \quad (7.37a)$$

$$[B] = -\frac{1}{L^2} [(6 - 12L_1) L(2L_2 - 4L_1) (6 - 12L_2) L(4L_2 - 2L_1)] \quad (7.37b)$$

The stiffness matrix for the beam element is given by

$$[k_m] = \int [B]^T [C] [B] dl \quad (7.38)$$

$$[k_m] = \frac{EI_z}{4} \int_0^L \begin{bmatrix} (6 - 12L_1)^2 & L(6 - 12L_1)(2L_2 - 4L_1) & (6 - 12L_1)(6 - 12L_2) \\ L^2(2L_2 - 4L_1)^2 & L(2L_2 - 4L_1)(6 - 12L_2) & (6 - 12L_2)^2 \\ (6 - 12L_2)^2 & L(6 - 12L_1)(4L_2 - 2L_1) & L^2(2L_2 - 4L_1)(4L_2 - 2L_1) \\ L(6 - 12L_2)(4L_2 - 2L_1) & L^2(4L_2 - 2L_1)^2 & L^2(4L_2 - 2L_1)^2 \end{bmatrix} dl \quad (7.39)$$

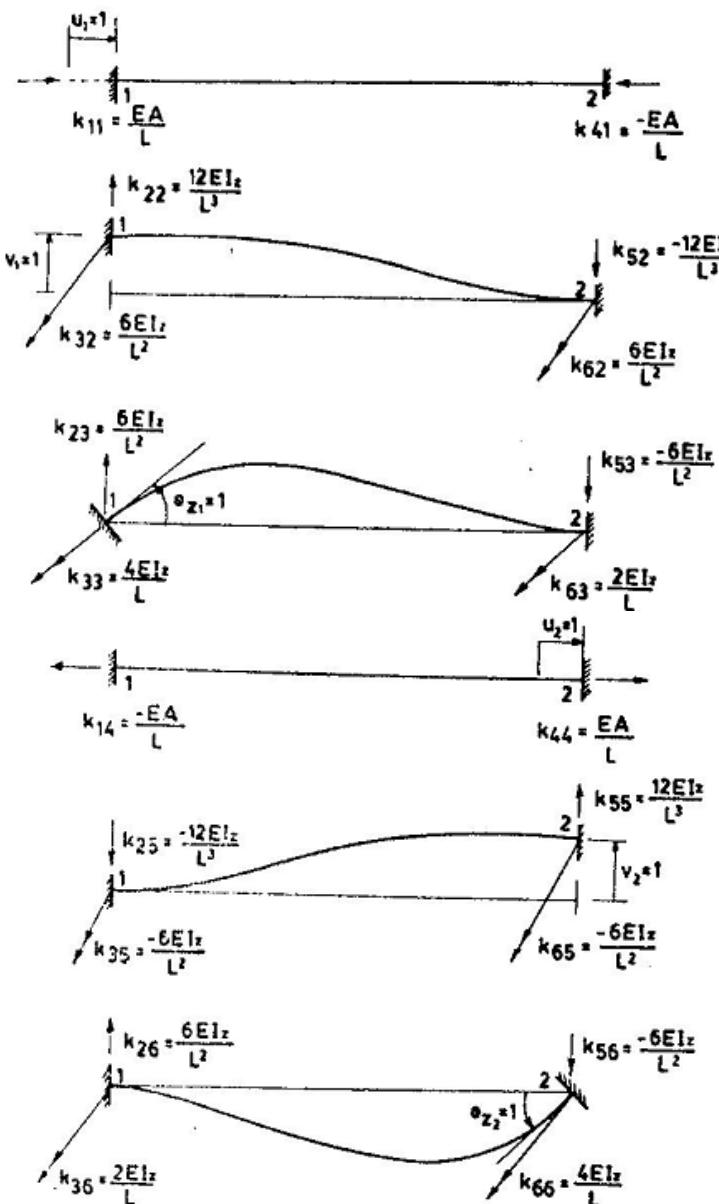
sym.

$$[k_m] = \begin{bmatrix} \frac{12EI_z}{L^3} & \frac{6EI_z}{L^2} & \frac{-12EI_z}{L^3} & \frac{6EI_z}{L^2} \\ & \frac{4EI_z}{L} & \frac{-6EI_z}{L^2} & \frac{2EI_z}{L} \\ \text{sym.} & & \frac{12EI_z}{L^3} & \frac{-6EI_z}{L^2} \\ & & & \frac{4EI_z}{L} \end{bmatrix} \quad (7.40)$$

$$\{d_m\}^T = [d_{1m}, d_{2m}, d_{3m}, d_{4m}, d_{5m}, d_{6m}] \quad (7.41)$$

2 Dim, 6 d.o.f

referred to member axes.



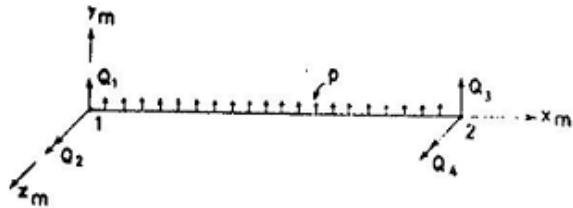
$$[k_m] = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI_z}{L^3} & \frac{6EI_z}{L^2} & 0 & -\frac{12EI_z}{L^3} & \frac{6EI_z}{L^2} \\ 0 & \frac{6EI_z}{L^2} & \frac{4EI_z}{L} & 0 & -\frac{6EI_z}{L^2} & \frac{2EI_z}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI_z}{L^3} & -\frac{6EI_z}{L^2} & 0 & \frac{12EI_z}{L^3} & -\frac{6EI_z}{L^2} \\ 0 & \frac{6EI_z}{L^2} & \frac{2EI_z}{L} & 0 & -\frac{6EI_z}{L^2} & \frac{4EI_z}{L} \end{bmatrix}$$

Computation of Element Nodal Load Vector

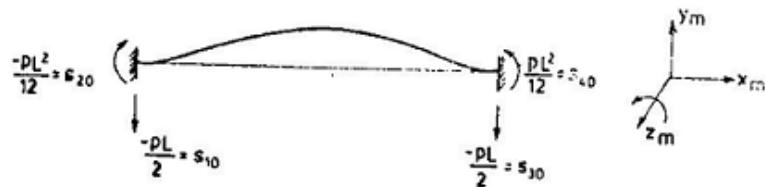
$$\{Q\} = \iint [N^s]^T \{p\} \, ds$$

$$\{Q\} = p \int [N^s]^T \, dl$$

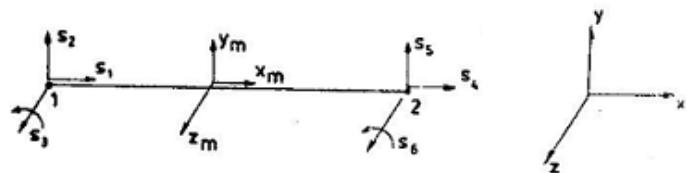
$$\{Q_m\} = \{Q\} = p \int_0^L \left\{ \begin{array}{l} L_1^2 (3 - 2L_1) \\ L_1^2 L_2 L \\ L_2^2 (3 - 2L_2) \\ - L_1 L_2^2 2L \end{array} \right\} \, dl = \left\{ \begin{array}{l} \frac{pL}{2} \\ \frac{pL^2}{12} \\ \frac{pL}{2} \\ -\frac{pL^2}{12} \end{array} \right\}$$



(a) NODAL LOAD VECTOR COMPONENTS



(b) FIXED END ACTIONS



(c) STRESS RESULTANTS

Fig. 7.9 Nodal load vector and stress resultant of a two dimensional beam member

Transformation Matrix

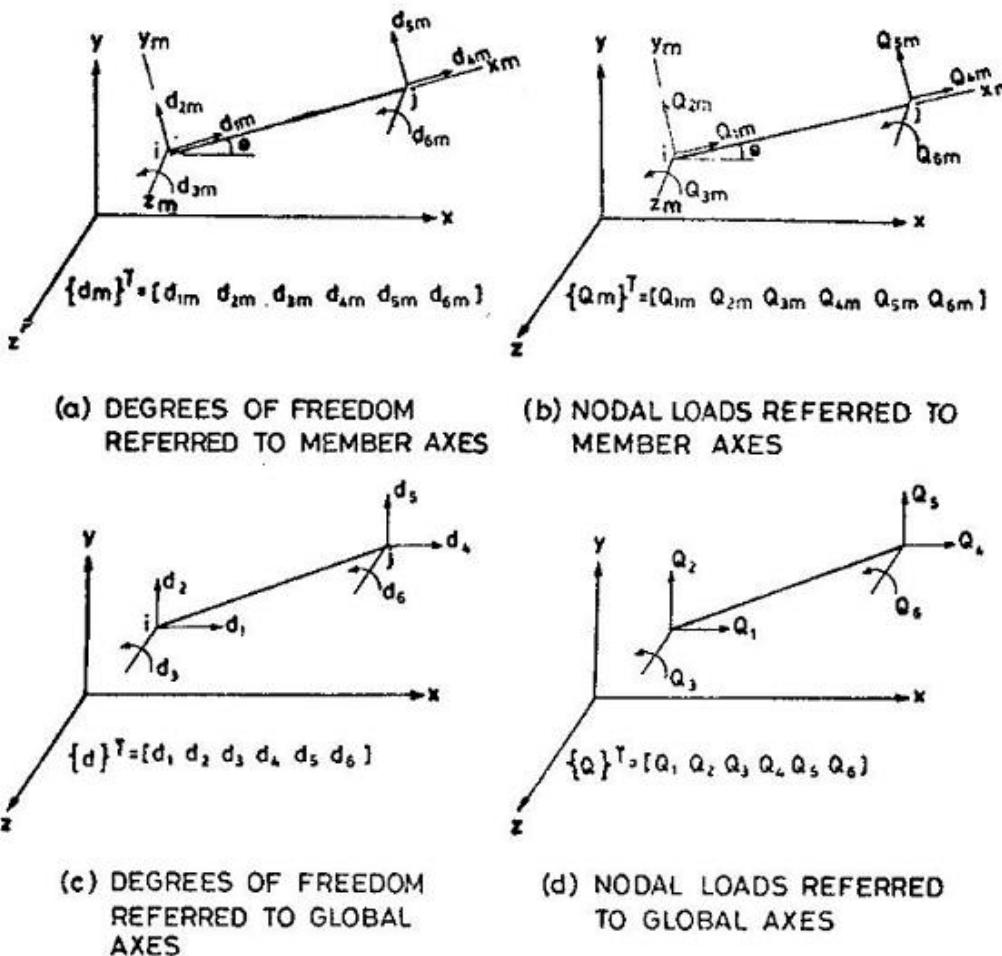


Fig. 7.10 Two-dimensional beam element

$$Q_{1m} = Q_1 \cos \theta + Q_2 \sin \theta = Q_1 C_x + Q_2 C_y$$

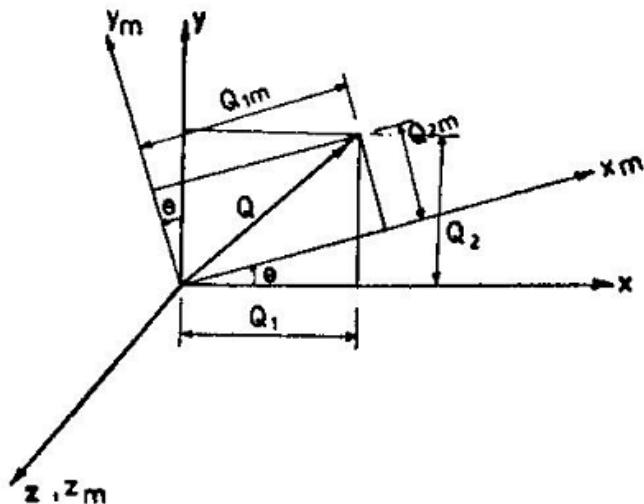
$$Q_{2m} = -Q_1 \sin \theta + Q_2 \cos \theta = -Q_1 C_y + Q_2 C_x \quad (7.45)$$

$$C_x = \cos \theta \text{ and } C_y = \sin \theta \quad (7.45a)$$

$$\begin{Bmatrix} Q_{1m} \\ Q_{2m} \\ Q_{3m} \end{Bmatrix} = \begin{bmatrix} C_x & C_y & 0 \\ -C_y & C_x & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{Bmatrix}$$

$$\{Q'_m\} = [T']^{-1} \{Q'\}$$

$$[T']^T = [T']^{-1}$$



$$\{Q'\} = [T']^T \{Q_m\}$$

$$\{d'_m\} = [T'] \{d'\}$$

$$\{d'\} = [T']^T \{d'_m\}$$

Fig. 7.11 Rotation of axes

$$\left\{ \begin{array}{c} Q_{1m} \\ Q_{2m} \\ Q_{3m} \\ Q_{4m} \\ Q_{5m} \\ Q_{6m} \end{array} \right\} = \left[\begin{array}{cccccc} C_x & C_y & 0 & 0 & 0 & 0 \\ -C_y & C_x & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & C_x & C_y & 0 \\ 0 & 0 & 0 & -C_y & C_x & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array} \right] \left\{ \begin{array}{c} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \end{array} \right\} \quad (7.51)$$

or

$$\left\{ \begin{array}{c} \{Q'_m\}_1 \\ \{Q'_m\}_2 \end{array} \right\} = \left[\begin{array}{cc} [T'] & [0] \\ [0] & [T'] \end{array} \right] \left\{ \begin{array}{c} \{Q'\}_1 \\ \{Q'\}_2 \end{array} \right\} \quad (7.52)$$

i.e.

$$\{Q_m\} = [T] \{Q\} \quad (7.53)$$

Similarly the nodal displacements can be related as

$$\{d_m\} = [T] \{d\} \quad (7.54)$$

And the inverse relations can also be expressed as

$$\{Q\} = [T]^T \{Q_m\} \quad (7.55)$$

and

$$\{d\} = [T]^T \{d_m\} \quad (7.56)$$

Now the equilibrium equation for the element in local system of axes can be expressed as

$$[k_m] \{d_m\} = \{Q_m\} \quad (7.57)$$

Substituting from Eqs. 7.53 and 7.54 into Eq. 7.57 we get

$$[k_m] [T] \{d\} = [T] \{Q\} \quad (7.58)$$

The vectors in Eq. 7.58 refer to the displacements and nodal forces in global system of axes. Premultiplying both sides of the equation by $[T]^{-1}$ which is equal to $[T]^T$, we get

$$[T]^T [k_m] [T] \{d\} = \{Q\} \quad (7.59)$$

This equation relates the displacements and the nodal forces in global axes.

Thus, $[k] \{d\} = \{Q\} \quad (7.60)$

where $[k] = [T]^T [k_m] [T]$ is the stiffness matrix of the element in the global axes system and is obtained by transforming $[k_m]$, the stiffness matrix in the member axes system.

Similarly, the nodal loads due to loads acting on the element can be transformed to the global system using Eq. 7.55 as

$$\{Q\} = [T]^T \{Q_m\} \quad (7.61)$$

Example 2 Analyse the continuous beam shown in Fig. 7.12 and draw the shear force and bending moment diagrams. The spring stiffness = $\frac{24EI}{L^3}$ and $EI=400$ units.

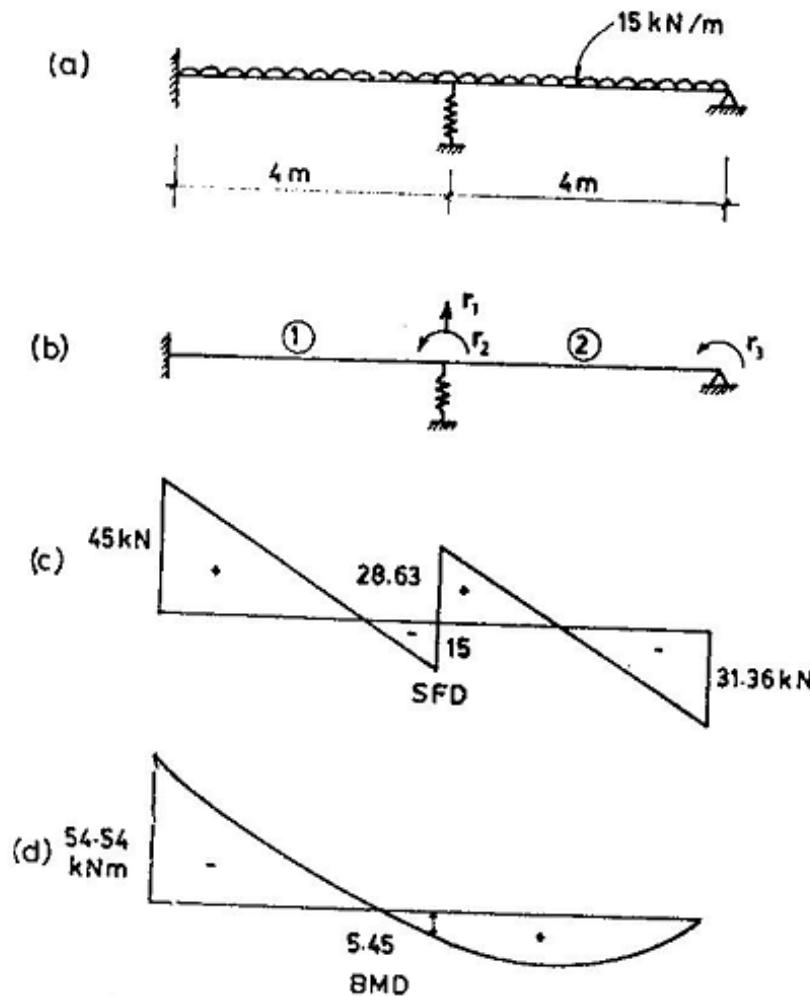


Fig. 7.12 Two span continuous beam

Table 7.1 Fixed-end actions for beam element

 $S_{30} = \frac{Wa}{L^2}$ $S_{60} = -\frac{Wb^2}{L^2}$ $S_{20} = \frac{Wb^2}{L^3}(3a+b)$ $S_{50} = \frac{Wa^2}{L^3}(a+3b)$	 $S_{30} = -S_{60} = \frac{WL}{12}$ $S_{20} = S_{50} = \frac{W}{2}$
 $S_{30} = \frac{Mb}{L^2}(2a-b)$ $S_{60} = \frac{Ma}{L^2}(2b-a)$ $S_{20} = -S_{50} = \frac{6Mab}{L^3}$	 $S_{30} = \frac{pL^2}{30}$ $S_{60} = -\frac{pL^2}{20}$ $S_{20} = \frac{3pL}{20}$ $S_{50} = \frac{7pL}{20}$
 $S_{10} = -\frac{Wb}{L}$ $S_{40} = -\frac{Wa}{L}$	 $S_{70} = \frac{M_x b}{L}$ $S_{80} = \frac{M_x a}{L}$

Member 1	g	d	o	f
	0	0	1	2
e	d	o	f	
	1	2	3	4

$$[k_1] = \begin{bmatrix} 0 & 1 & 75 & 150 & -75 & 150 \\ 0 & 2 & & 400 & -150 & 200 \\ 1 & 3 & \text{sym.} & & 75 & -150 \\ 2 & 4 & & & -150 & 400 \end{bmatrix}$$

(a)

$$\{Q_1\} = -\{S_{01}\} = - \begin{Bmatrix} \frac{WL}{2} \\ -\frac{WL^2}{12} \\ -\frac{WL}{2} \\ -\frac{WL^2}{12} \end{Bmatrix} = \begin{array}{c|ccccc} & 0 & 1 & & -30 \\ & 0 & 2 & & -20 \\ & 1 & 3 & & -30 \\ & 2 & 4 & & 20 \end{array} \begin{Bmatrix} -30 \\ -20 \\ -30 \\ 20 \end{Bmatrix}$$

g e

d d

o o

f f

Member 2	g d o f		1	2	0	3
	e d o f		1	2	3	4

$$[k_2] = \begin{matrix} & & \\ & & \\ [k_2] = & \begin{matrix} 1 & 1 & \begin{bmatrix} 75 & 150 & -75 & 150 \\ & 400 & -150 & 200 \\ & \text{sym} & 75 & -150 \\ & 4 & & 400 \end{bmatrix} \\ 2 & 2 & \\ 0 & 3 & \\ 3 & 4 & \end{matrix} \end{matrix}$$

g e

d d

o o

f f

$$\{Q_2\} = -\{S_{02}\} = \begin{matrix} & & \\ & & \\ [Q_2] = & \begin{matrix} 1 & 1 & \begin{Bmatrix} -30 \\ -20 \\ -30 \\ -20 \end{Bmatrix} \\ 2 & 2 & \\ 0 & 3 & \\ 3 & 4 & \end{matrix} \end{matrix}$$

The spring at the mid support has stiffness = $\frac{24 \times 400}{4^3} = 150 \text{ kN/m}$

Assembling the stiffness matrix and the load vector for the structure we have the equation of equilibrium for the continuous beam as

$$\begin{bmatrix} 300 & 0 & 150 \\ & 800 & 200 \\ \text{sym.} & & 400 \end{bmatrix} \begin{Bmatrix} r_1 \\ r_2 \\ r_3 \end{Bmatrix} = \begin{Bmatrix} -60 \\ 0 \\ 20 \end{Bmatrix} \quad (\text{e})$$

Solving the above equations we get,

$$r_3 = \frac{2}{11}$$

$$r_2 = -\frac{1}{22}$$

$$r_1 = -\frac{16}{55} \quad (\text{f})$$

The final stress resultants for each member are calculated using Eq. 7.63.

Member 1

$$\begin{bmatrix} 75 & 150 & -75 & 150 \\ & 400 & -150 & 200 \\ \text{sym.} & & 75 & -150 \\ & & & 400 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ -16/55 \\ -1/22 \end{Bmatrix} + \begin{Bmatrix} 30 \\ 20 \\ 30 \\ -20 \end{Bmatrix} = \begin{Bmatrix} 45.0 \\ 54.54 \\ 15.0 \\ 5.45 \end{Bmatrix} \quad (\text{g})$$

Member 2

$$\begin{Bmatrix} 75 & 150 & -75 & 150 \\ & 400 & -150 & 200 \\ \text{sym.} & & 75 & 150 \\ & & & 400 \end{Bmatrix} \begin{Bmatrix} -\frac{16}{55} \\ -\frac{1}{22} \\ 0 \\ \frac{2}{11} \end{Bmatrix} + \begin{Bmatrix} 30 \\ 20 \\ 30 \\ -20 \end{Bmatrix} = \begin{Bmatrix} 28.63 \\ -5.45 \\ 31.36 \\ 0.0 \end{Bmatrix}$$

Example 3 Analyse the plane frame shown in Fig. 7.13 by using the 2D beam element. Draw the bending moment diagram.

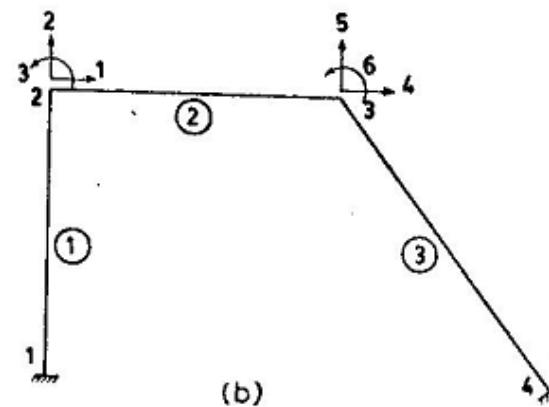
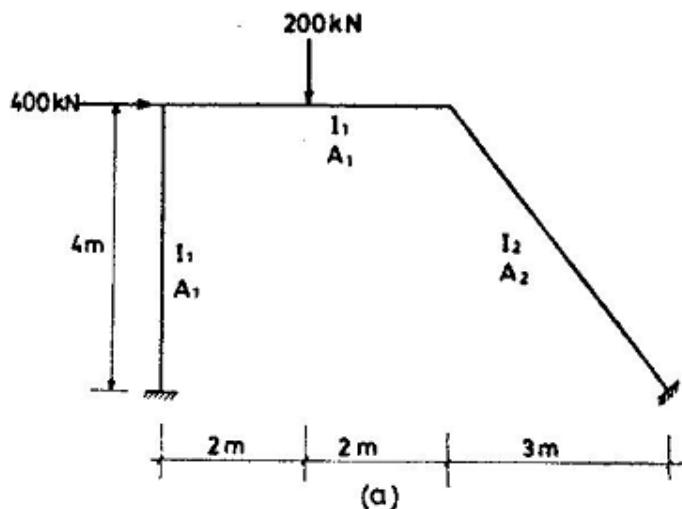
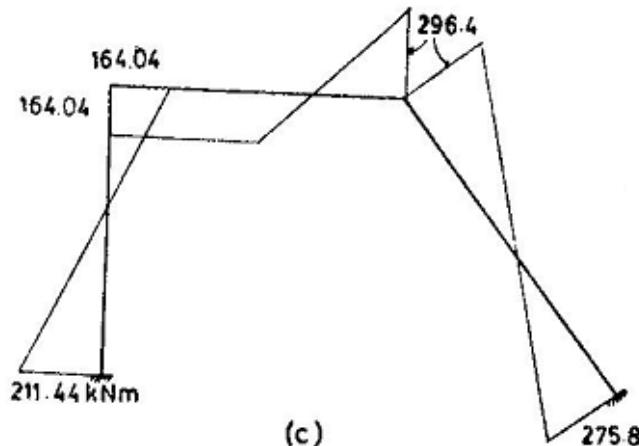


Fig. 7.13 Plane frame



$$E = 2 \times 10^7 \text{ kN/m}^2, I_1 = 12 \times 10^{-5} \text{ m}^4, A_1 = 0.03 \text{ m}^2$$

$$I_2 = 15 \times 10^{-5} \text{ m}^4, A_2 = 0.035 \text{ m}^2$$

Using the transformation matrix $[T]$ given by Eq. 7.51, the stiffness matrix $[k]$ of the member in the global axes system is computed from Eq. 7.60. Similarly the element nodal load vectors $\{Q_m\}$ and $\{Q\}$ are calculated using the procedure indicated in subsection 7.3.4 and the Eq. (7.55). The numerical values for the members 1, 2 and 3 are given below.

$$[k_m] = \begin{bmatrix} 15 \times 10^4 & 0 & 0 & -15 \times 10^4 & 0 & 0 \\ 0 & 450 & 900 & 0 & -450 & 900 \\ 0 & 900 & 2400 & 0 & -900 & 1200 \\ -15 \times 10^4 & 0 & 0 & 15 \times 10^4 & 0 & 0 \\ 0 & -450 & -900 & 0 & 450 & -900 \\ 0 & 900 & 1200 & 0 & -900 & 2400 \end{bmatrix} \quad (a)$$

$$C_x = 0, C_y = 1.0$$

$$[T] = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (b)$$

The stiffness matrix $[k]$ in global axes is given by,

$$[k_1] = [T]^T [k_m] [T]$$

$$[k_1] = \begin{bmatrix} g & d & o & f & 0 & 0 & 0 & 1 & 2 & 3 \\ e & d & o & f & 1 & 2 & 3 & 4 & 5 & 6 \\ 0 & 1 & 450 & 0 & -900 & -450 & 0 & -900 \\ 0 & 2 & 0 & 15 \times 10^4 & 0 & 0 & -15 \times 10^4 & 0 \\ 0 & 3 & -900 & 0 & 2400 & 900 & 0 & 1200 \\ 1 & 4 & -450 & 0 & 900 & 450 & 0 & 900 \\ 2 & 5 & 0 & -15 \times 10^4 & 0 & 0 & 15 \times 10^4 & 0 \\ 3 & 6 & -900 & 0 & 1200 & 900 & 0 & 2400 \\ g & e \\ d & d \\ o & o \\ f & f \end{bmatrix} \quad (c)$$

Member 2

Since the member is oriented in the global directions, no transformation is required. The member stiffness matrix in the local directions is the same as in the case of member 1.

Thus

$$[k_2] = \begin{bmatrix} g & d & o & f & 1 & 2 & 3 & 4 & 5 & 6 \\ e & d & o & f & 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 1 & 15 \times 10^4 & 0 & 0 & -15 \times 10^4 & 0 & 0 & 0 & 0 \\ 2 & 2 & 0 & 450 & 900 & 0 & -450 & 900 & 0 & 0 \\ 3 & 3 & 0 & 900 & 2400 & 0 & -900 & 1200 & 0 & 0 \\ 4 & 4 & -15 \times 10^4 & 0 & 0 & 15 \times 10^4 & 0 & 0 & 0 & 0 \\ 5 & 5 & 0 & -450 & -900 & 0 & 450 & -900 & 0 & 0 \\ 6 & 6 & 0 & 900 & 1200 & 0 & -900 & 2400 & 0 & 0 \end{bmatrix} \quad (d)$$

g e

The element load vector also does not require any transformation.

d d

o o

f f

$$\{Q_2\} = \left\{ \begin{array}{c} 0 \\ -\frac{WL}{2} \\ -\frac{WL}{8} \\ 0 \\ -\frac{WL}{2} \\ \frac{WL}{8} \end{array} \right\} = \left\{ \begin{array}{c} 0 \\ -100 \\ -100 \\ 0 \\ -100 \\ 100 \end{array} \right\} \quad (e)$$

Member 3

$$[k_m] = \begin{bmatrix} 14 \times 10^4 & 0 & 0 & -14 \times 10^4 & 0 & 0 \\ 0 & 288 & 720 & 0 & -288 & 720 \\ 0 & 720 & 2400 & 0 & -720 & 1200 \\ -14 \times 10^4 & 0 & 0 & 14 \times 10^4 & 0 & 0 \\ 0 & -288 & -720 & 0 & 288 & -720 \\ 0 & 720 & 1200 & 0 & -720 & 2400 \end{bmatrix}$$

(f)

$$C_x = 0.6, C_y = -0.8$$

$$[T] = \begin{bmatrix} 0.6 & -0.8 & 0 & 0 & 0 & 0 \\ 0.8 & 0.6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.6 & -0.8 & 0 \\ 0 & 0 & 0 & 0.8 & 0.6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(g)

$$[k_3] = [T]^T [k_m] [T]$$

g	d	o	f	4	5	6	0	0	0
e	d	o	f	1	2	3	4	5	6

$$[k_3] = \begin{bmatrix} 4 & 1 & 50584 & -67062 & 576 & -50584 & 67062 & 576 \\ 5 & 2 & -67062 & 89704 & 432 & 67062 & -89704 & 432 \\ 6 & 3 & 576 & 432 & 2400 & 576 & 432 & 1200 \\ 0 & 4 & -50584 & 67062 & 576 & 50584 & -67062 & -576 \\ 0 & 5 & 67062 & -89704 & 432 & -67062 & 89704 & -432 \\ 0 & 6 & 576 & 432 & 1200 & -576 & -432 & 2400 \end{bmatrix} \quad (h)$$

g	e								
d	d								
o	o								
f	f								

The stiffness contribution from each member is added to get the stiffness matrix of the structure following the procedure described in Chap. 5
 The equilibrium equation for the frame is given by,

$$\left[\begin{array}{cccccc} 150450 & 0 & 900 & -150000 & 0 & 0 \\ 0 & 150450 & 900 & 0 & -450 & 900 \\ 900 & 900 & 4800 & 0 & -900 & 1200 \\ -150000 & 0 & 0 & 200584 & -67062 & 576 \\ 0 & -450 & -900 & -67062 & 90154 & -468 \\ 0 & 900 & 1200 & 576 & -468 & 4800 \end{array} \right] \left\{ \begin{array}{l} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \end{array} \right\} = \left\{ \begin{array}{l} 400.0 \\ -100.0 \\ -100.0 \\ 0.0 \\ -100.0 \\ 100.0 \end{array} \right\} \quad (i)$$

Solving the equation (i) we get,

$$\{r\} = \left\{ \begin{array}{l} 0.2876 \\ 0.00010073 \\ -0.0395 \\ 0.2856 \\ 0.2107 \\ 0.0169 \end{array} \right\}$$

Member End Forces

The stress resultants at the ends of the members are calculated using the equation (7.63),

Member 1

$$\{d_m\} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0.2876 \\ 1.0073 \times 10^{-4} \\ -0.0395 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 1.0073 \times 10^{-4} \\ -0.2876 \\ -0.0395 \end{Bmatrix} \quad (k)$$

$$\{S\} = \begin{bmatrix} 15 \times 10^4 & 0 & 0 & -15 \times 10^4 & 0 & 0 \\ 0 & 450 & 900 & 0 & -450 & 900 \\ 0 & 900 & 2400 & 0 & -900 & 1200 \\ -15 \times 10^4 & 0 & 0 & 15 \times 10^4 & 0 & 0 \\ 0 & -450 & -900 & 0 & 450 & -900 \\ 0 & 900 & 1200 & 0 & -900 & 2400 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 1.0073 \times 10^{-4} \\ -0.2876 \\ -0.0395 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} -15.11 \\ 93.87 \\ 211.44 \\ 15.11 \\ -93.87 \\ 164.04 \end{Bmatrix} \quad (l)$$

Member 2

Since the member axes are parallel to the global axes, $\{d_m\} = \{d\}$

$$\{S\} = \begin{bmatrix} 15 \times 10^4 & 0 & 0 & -15 \times 10^4 & 0 & 0 \\ 0 & 450 & 900 & 0 & -450 & 900 \\ 0 & 900 & 2400 & 0 & -900 & 1200 \\ -15 \times 10^4 & 0 & 0 & 15 \times 10^4 & 0 & 0 \\ 0 & -450 & -900 & 0 & 450 & -900 \\ 0 & 900 & 1200 & 0 & -900 & 2400 \end{bmatrix} \begin{Bmatrix} 0.2876 \\ 1.0073 \times 10^{-4} \\ -0.0395 \\ 0.2856 \\ 0.2107 \\ 0.0169 \end{Bmatrix} + \begin{Bmatrix} 0.0 \\ 100.0 \\ 100.0 \\ 0.0 \\ 100.0 \\ -100.0 \end{Bmatrix} = \begin{Bmatrix} 300.0 \\ -15.1 \\ -164.06 \\ -300.0 \\ 215.1 \\ -296.4 \end{Bmatrix} \quad (m)$$

Member 3

$$\{d_m\} = \begin{bmatrix} 0.6 & -0.8 & 0 & 0 & 0 & 0 \\ 0.8 & 0.6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.6 & -0.8 & 0 \\ 0 & 0 & 0 & 0.8 & 0.6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} 0.2856 \\ 0.2107 \\ 0.0169 \\ 0.0 \\ 0.0 \\ 0.0 \end{Bmatrix} = \begin{Bmatrix} 2.8 \times 10^{-3} \\ 0.3549 \\ 0.0169 \\ 0.0 \\ 0.0 \\ 0.0 \end{Bmatrix} \quad (n)$$

$$\{S\} = \begin{bmatrix} 14 \times 10^4 & 0 & 0 & -14 \times 10^4 & 0 & 0 \\ 0 & 288 & 720 & 0 & -288 & 720 \\ 0 & 720 & 2400 & 0 & -720 & 1200 \\ -14 \times 10^4 & 0 & 0 & 14 \times 10^4 & 0 & 0 \\ 0 & -288 & -720 & 0 & 288 & -720 \\ 0 & 720 & 1200 & 0 & -720 & 2400 \end{bmatrix} \times \begin{Bmatrix} 2.8 \times 10^{-3} \\ 0.3549 \\ 0.0169 \\ 0.0 \\ 0.0 \\ 0.0 \end{Bmatrix} = \begin{Bmatrix} +392.0 \\ +114.4 \\ 296.1 \\ -392.0 \\ -114.4 \\ 275.8 \end{Bmatrix} \quad (o)$$

The final bending moment diagram for the frame is shown in Fig. 7.13c.

